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LIFE ANALYSIS OF RESTORED AND REFURBISHED BEARINGS

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16. Abstract An analysis of the potential life of refurbished and restored bearings was performed. The sensitivity of 10-percent life and mean-time-between-failure to the effects of cumulative fatigue damage and the amount of stressed volume removed in the restoration process were examined. A modified Lundberg-Palmgren theory was used to predict that the expected 10-percent life of a restored bearing, which is dependent on the previous service time and the volume of material removed from the race surfaces, can be between 74 and 100 percent of the new bearing life. Using renewal theory, we found that the mean time between failure ranged from 90 to 100 percent of that for a new bearing.					
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SUMMARY

A life analysis for refurbished and restored bearings was performed. The analysis was based on a modification of the Lundberg-Palmgren theory for rolling-element bearings. The Lundberg-Palmgren equation was modified to show the effect of removing (by grinding) a portion of the stressed volume of the bearing. The theory shows that for the refurbished and restored bearings there is a bias toward more early failures. Depending on the amount of material removed in the grinding process and depending on the time at which the bearing is removed from service to be restored, the 10-percent life of the refurbished and restored bearings ranges from 74 to 100 percent of the 10-percent life of a brand new bearing.

Renewal theory was used to predict the expected mean time between failure (MTBF). The calculations were performed using a digital computer. The MTBF for refurbished and restored bearings ranges from 90 to 100 percent of the MTBF for brand new bearings. Since this is not a significant difference, existing maintenance and replacement policies should not be adversely affected by the use of restored and refurbished bearings for replacements at overhaul time.

INTRODUCTION

During the last three decades the severity of the conditions under which rolling-element bearings are expected to function reliably and with long life have increased significantly. Rolling-element bearings are now required to operate at much higher speeds and somewhat higher temperatures than in the early 1940's. The increased speed and temperature requirements originated principally with the advent of the aircraft gas-turbine engine. Its development, coupled with the appearance of a variety of high-speed turbine-driven machines, has resulted in a wide range of rolling-element bearing requirements for mainshaft, accessory, and transmission applications.

Classical rolling-element fatigue, which is of subsurface origin has been considered

the primary life limiting factor for rolling-element bearings, although actually less than 10 percent of them fail by fatigue (ref. 1). Even under the ideal conditions of proper design, handling, installation, lubrication, and system cleanliness, a rolling-element bearing will eventually fail by fatigue. Because fatigue results from inherent material weaknesses, research to improve material quality is a continuing activity. The remaining 90 percent of the failures are due to causes such as lubricant flow interruption, lubricant contamination, lubricant deterioration, excessive dirt ingestion, improper bearing installation, incorrect mounting fits, mishandling of bearings before installation, contaminated bearings, manufacturing defects, ring growth in service, and corrosion (ref. 2). These causes can result in as many as 50 percent of the bearings being rejected at overhaul time. In most cases the remaining fatigue life of the rejected bearings is many times the accumulated hours already on the bearing.

A major concern facing military and civilian aviation is the replacement cost of component parts such as rolling-element bearings. Rolling-element bearings are designed for long-term operation in an engine or transmission application. The design life for these bearings is based on the surface pitting fatigue criteria. Scrapped bearings other than those which fail from fatigue, are generally surface damaged on the raceways or are out of blueprint tolerance. Current bearing rework methods, unfortunately, cannot save these bearings and at the same time assure the reliability required for aviation applications (ref. 3).

It has been proposed that bearings removed at overhaul be reused after refurbishment or restoration. Refurbished bearings are disassembled, cleaned, and visually and dimensionally inspected. If no major imperfections are found, the bearings are reassembled, lubricated, and packaged for further service. In some cases new rolling-elements are inserted (ref. 3).

Bearing restoration represents a logical extension of the practice of bearing refurbishment. Restoration entails grinding the races and other critical surfaces of used bearings to their original characteristics and dimensions. Grinding can remove raceway imperfections to significant depths below the surface. In addition, this stock removal is accomplished by grinding races on the same machines and with the same controls that are used in the manufacture of new bearing raceways. The result is geometrical accuracy and surface finished identical to new bearing raceways (ref. 4).

The objectives of the work reported herein were (1) to determine analytically the additional life potential of refurbished or restored rolling-element bearings, and (2) to predict the replacement rate for restored bearings. In order to accomplish the aforesaid, a modified Lundberg-Palmgren analysis (refs. 5 to 7) was performed on bearings subjected to restoration. Using statistical techniques similar to those of reference 8, a renewal theory was developed for restored rolling-element bearings. The results of this analysis were compared with the replacement rate for new bearings.

SYMBOLS

a	semiwidth of Hertzian contact ellipse
c	exponent
e	Weibull slope
F	probability of failure
f	failure probability density function
h	exponent
L_{10}	10-percent life of brand new bearing
l	length of rolling track
$P(x)$	probability for occurrence of event x
q	maximum Hertzian contact stress
R	cumulative renewal function
R_{10}	10-percent life of refurbished and restored bearing
r	renewal density function
s	probability of survival
T	dummy variable of integration
t	time
V	stressed volume
x	fraction of stressed volume removed
z	depth under surface
z_0	depth to maximum orthogonal reversing shear stress
η	number of stress cycles
μ	mean time to failure
τ	subsurface orthogonal reversing shear stress
τ_0	maximum value of τ

Subscripts:

n	new material
r	refurbished and restored bearings
t	time of restoration
u	used material
0	corresponding to $\eta_r = 0$

RESTORATION PROCEDURE

Pitting fatigue is caused by the cyclic application of loads on the rolling elements and races as the bearing rotates. Lundberg and Palmgren (ref. 5) assumed that most fatigue damage originates below the surface of the material and generally runs in planes parallel to the direction of rolling. Therefore, they directed their attention to the orthogonal reversing shear stress, which occurs on planes parallel to the direction of rolling. The magnitude of the orthogonal reversing shear stress is plotted as a function of depth beneath the surface in figure 1. Much the same role is played by the subsurface maximum shear stress on a 45° plane, when the fatigue cracks originate in that direction.

Microscopically small weak spots or defects in the bearing steel, such as small discontinuities, play an essential role in the initiation of a fatigue crack. In a homogenous material the weak spots are uniformly dispersed throughout the total material volume. The magnitude of the material volume that is stressed is a measure of the number of weak spots that may be sites for the initiation of fatigue failure. As the total stressed volume of material is increased, the probability of fatigue failure increases, since there is a greater likelihood of a weak spot coinciding with a high shear stress. The stressed volume, which contains the zone of high shearing stresses, extends approximately 0.15 millimeter (0.006 in.) beneath the surface for most rolling-element bearing applications. As an expression for the magnitude of the stressed volume, we use (as did Lundberg and Palmgren (ref. 5)) the expression az_0l , where a is the semiwidth of the Hertzian contact ellipse, z_0 is the depth of occurrence of the maximum orthogonal reversing shear stress, and l is the length of the rolling contact path.

When a bearing raceway is damaged by fatigue spalling, it is not considered for restoration by grinding. If, however, there is superficial damage to the bearing raceways, caused by dirt or fatigue debris from the rolling elements, raceways can often be restored by grinding. In general superficial damage extends less than 0.05 millimeter (0.002 in.) from the surface.

The process of bearing restoration by grinding was first reported in references 1 and 4. Rejected bearings are disassembled, the components are visually inspected, and the hardnesses of the bearing races are measured. The components that are determined to be restorable are dimensionally inspected. Where necessary, the bearing faces, bores, and outer diameters are ground and either nickel or chrome plated to a thickness that will allow the surfaces to be reground to the original blueprint dimensions. Both inner and outer raceways are ground to a depth of at least 0.05 millimeter (0.002 in.) but not more than 0.15 millimeter (0.006 in.), which removes all superficial damage and a large portion of the fatigue damaged stressed volume. The surface is finished to its original blueprint specification. The bearing is then refitted with new rolling elements having a diameter equal to the diameter of the elements previously contained in the bear-

ing plus twice the depth of regrinding. The bearing separator is stripped of its silver plating, where applicable, inspected for cracks, and replated. The new, rolling elements are placed within the separator, and the bearing is reassembled.

For the cylindrical roller bearings the procedure is the same except that the roller length as well as the roller diameter are increased by a value of twice the depth of regrinding.

For ball bearings the effective race curvature is the same as the original dimensions within significant mathematical values. The original values of contact angle, resting angle, and radial clearance are the same. Although the restored bearing contains oversize balls and oversize raceways, the total effective geometry of the bearing has not been changed, and consequently, the contact stress level will be essentially identical to that of the original bearing.

ANALYSIS

Failure Theory

The accepted method for calculating rolling-element bearing fatigue life is based on the Lundberg-Palmgren life formula (refs. 5 to 7). This formula describes the functional relations among the probability of survival, the number of stress cycles to failure, stress, stressed volume, and depth of occurrence of the maximum orthogonal reversing shear stress. The statistical dispersion in life satisfies the Weibull distribution as follows:

$$\log \frac{1}{S} \propto \frac{\tau_o^c \eta^e}{z_o^h} V \quad (1)$$

In the present case a comparison of the life of restored or refurbished bearings to the life of brand new bearings is required. For the purpose of comparing the failure distributions of the two groups, a constant of proportionality is not required in equation (1).

The basic failure theory will now be applied to analyzing the reliability of refurbished and restored bearings. Let it be assumed that when refurbished and restored bearings are put back into service they are run under the original service conditions. The maximum value of the orthogonal reversing shear stress is unchanged, but its location is shifted deeper into the original bearing material according to the amount of material removed during grinding. (See fig. 1.) Therefore, the stressed volume of a restored bearing is composed of a newly stressed portion and an older portion with a history of stress cycles on it. The probability of survival of the restored bearing is

equal to the product of the probabilities of survival of the new material with that of the used material.

$$S_r = S_n S_u \quad (2)$$

The removal of a material layer of the stressed volume in the fractional amount x exposes a new layer of material. The probability of survival for this layer is based on equation (1):

$$S_n = \exp \left[-x \left(\frac{\eta_r}{L_{10}} \right)^e \ln \frac{1}{0.9} \right] \quad (3)$$

where L_{10} is the 10-percent life of the original bearing and η_r is the additional number of stress cycles after restoration.

Next, consider the used material portion of the stressed volume. If the cycles of stress accumulated before restoration are denoted by η_t , then the following equation gives the probability of survival for the used portion of the material in the restored bearing:

$$S = \exp \left[- (1 - x) \left(\frac{\eta_t + \eta_r}{L_{10}} \right)^e \ln \frac{1}{0.9} \right] \quad (4)$$

The probability of survival of the used material is S_0 when $\eta_r = 0$ (i. e., as soon as the bearing is started in operation again). However, $S_0 < 1$ according to equation (4). This means that there is a finite probability of failure F_0 immediately on starting the restored bearing:

$$F_0 = 1 - S_0 \quad (5)$$

The physical reasons for this are simply that the material has already endured η_t stress cycles and that some of the bearings have been damaged. However, it must be assumed that such bearings will be detected during inspection and scrapped. The normalized probability of failure, which includes the effect of having discarded the bearings expected to immediately fail due to incipient cracks, is given by the following equation:

$$F_u = \frac{F - F_0}{S_0} \quad (6)$$

The expressions

$$S_u = 1 - F_u \quad (7a)$$

$$S = 1 - F \quad (7b)$$

and equations (5) and (6) may be used to write the normalized probability of survival for the used material as follows:

$$S_u = \frac{S}{S_0} \quad (8)$$

From equations (2) to (4) and (8) the probability of survival for the restored bearing is written as a function of the time at which restoration occurs and fraction of stressed volume removed.

$$S_r = \exp\left(\left(\ln \frac{1}{0.9}\right)\left\{(x-1)\left[\left(\frac{\eta_t + \eta_r}{L_{10}}\right)^e - \left(\frac{\eta_t}{L_{10}}\right)^e\right] - x\left(\frac{\eta_r}{L_{10}}\right)^e\right\}\right) \quad (9)$$

Equation (9) gives the probability of survival that would be expected if an endurance test were run on a group of restored bearings.

Bearing Replacement Rate

Endurance testing is a valuable tool for groups of bearings manufactured using different processes or different materials; in the present context it would aid in determining the effect of the restoration on bearing fatigue life. In a typical endurance test the time to failure or time to suspension of test for each bearing is recorded. Bearings that fail are not replaced. The laboratory test is concluded when all of the test group have been used.

Although the laboratory tests are often conducted consecutively because of a limited number of test machines, the results can be interpreted as if the bearings were run concurrently. The laboratory results would describe the failure rate in field service only if all bearings were put into service at the same time and if failed bearings were not replaced. But, if a certain number of bearings must be kept in operation, failure rate with replacements is of interest. For example, if there are 10 000 bearings in the field, the maintenance and supply manager must know how many spare bearings will be needed

over a given period in order to keep 10 000 bearings in operation at all times. Over the interval of service operation it may take 15 000 replacements to keep all 10 000 bearings running. Thus, laboratory failures can never exceed 100 percent, but field failures can and often do exceed 100 percent.

The expected failure rate in the field can be mathematically related to laboratory test results using renewal theory similar to that presented in reference 8. If a bearing failure occurs in the field it may be one of the original bearings or it may be one of the replacement bearings. In fact, a given failure occurrence could be due to failure of the second, third, or later replacement bearing.

The cumulative probability of failure for the first time, assuming constant service conditions, is a function of the length of time that the bearing has been running. Failure is the complimentary function to survival according to the following relation:

$$F(t) = 1 - S(t) \quad (10)$$

The probability density function for failure is

$$f(t) \equiv \frac{dF}{dt} \quad (11)$$

The instantaneous probability of failure for the first time in any time interval from t to $t + \Delta t$ is then given by

$$P(\text{first failure in } \Delta t \text{ interval}) = f(t)\Delta t \quad (12)$$

By using renewal theory (refs. 11 and 12), the probability of having to replace a bearing in the field is written as follows:

$$P(\text{making a replacement}) = r(t)\Delta t \quad (13)$$

where $r(t)$ is called the renewal density. The renewal density is calculated from the following summation (ref. 11):

$$r(t) = \sum_{k=1}^{\infty} f_k(t) \quad (14)$$

where $f_k(t)$ is the k -fold convolution of f with itself and is computed by the following recurrence relation (ref. 11):

$$f_{k+1}(t) = \int_0^t f_k(T)f_1(t-T)dT \quad (f_1 \equiv f) \quad (15)$$

The expression $f_k(t)\Delta t$ gives the probability of a k^{th} failure occurring in the time interval t to $t + \Delta t$. Since, when a failure occurs, it can be for any value of k , it follows that the renewal density function should be defined as the sum over all the f_k 's.

The total number of replacements made during the first t units of time is obtained by integrating the renewal density function as follows:

$$R(t) = \int_0^t r(T)dT \quad (16)$$

For a group of bearings that has been operating for some time with failures occurring and replacements being made, it is also important to know the MTBF (mean time expected between failures). According to the renewal density theorem (ref. 12),

$$\lim_{t \rightarrow \infty} r(t) = \frac{1}{\int_0^\infty tf(t)dt} \equiv \frac{1}{\mu} \quad (17)$$

Therefore, the mean time between failures is calculated as

$$\text{MTBF} = \frac{\mu}{\text{total number of bearings in system}} \quad (18)$$

RESULTS AND DISCUSSION

Parametric results were generated using equation (9). For computation two restoration times were arbitrarily chosen: the 50-percent life ($\eta_t = L_{50}$) and the 10-percent life ($\eta_t = L_{10}$). The results are plotted on Weibull coordinates (fig. 2) with the fraction of stressed volume removed during restoration as a parameter. Weibull coordinates (ref. 9) are the log-log of the reciprocal of the probability of survival plotted as the statistical percentage of specimens failed (ordinate) against the log of time to failure (abscissa). The calculations were performed for $x = 0, 1/4, 1/2, 3/4$, and 1, assuming a Weibull slope of 10/9.

The extreme values of $x = 1$ and $x = 0$ are special cases. For $x = 1$ the entire stressed volume layer is removed, and, therefore, an essentially brand new bearing is the result. The failure rate in this case is Weibull in nature and plots as a straight

line on Weibull coordinates. For $x = 0$ no grinding is performed; the failed or damaged bearings are merely culled from the total population at time η_t . (This condition is that of a refurbished bearing.) The remaining bearings are put into service again. The results for $x = 0$ agrees with the result presented in reference 10 for this special case.

For values other than $x = 1$ the failure rate is non-Weibull because the plot is non-linear. There are proportionately more early failures than would be the case for a true Weibull distribution. In general, for increasing amounts of material removed in restoration, the theory predicts closer approximation to the new bearing failure rate. The theory also shows that if a given amount of material is to be removed in restoration, then longer bearing life will be achieved with an earlier refurbishment, say at the 10-percent life rather than the 50-percent life.

The ratio of the 10-percent life of restored bearings to the 10-percent life of brand new bearings is shown in figure 3. As would be expected, the shortest life conditions occur when the bearings are restored at the 50-percent level with a minimum amount of stressed volume removed.

A computer program was written to evaluate equation (14). Calculations were performed using $x = 0, 1/4, 1/2, 3/4$, and 1 and assuming a Weibull slope of 10/9. Figure 4 shows plotted results, which give the renewal density function for the limiting cases of $x = 0$ and 1. Restoration at the 10-percent level and the 50-percent level were considered. For comparison, the probability density for failures with no replacement (Weibull density function) is plotted also. The area under the curves represents the probability of failure.

The functions plotted in figure 4 were numerically integrated and are shown plotted in figure 5. These are the cumulative functions for renewal or failure. The cumulative renewal functions indicate that 100 percent replacement bearings will have been needed by the time that 8 L_{10} intervals have elapsed. By comparison, at dimensionless time of 8, the Weibull distribution shows that only 65 percent of the original bearings will have failed. The difference in total failures would be due to replacement bearings failing.

Figure 6 shows how the mean time between failure is affected by restoration time η_t and by fraction of stressed volume removed x . As would be expected, the results show that the restored bearings have longer MTBF when more material is removed ($x \approx 1$) and when the restoration is performed earlier ($\eta_t \approx L_{10}$). A comparison of figures 6 and 3 shows that the MTBF is not as sensitive as the R_{10} life to parametric changes in η_t and x . In other words, the disparity in life between brand new bearings and restored bearings is much more noticeable in endurance testing than in field usage. If the worst conditions ($\eta_t = 50, x = 0$) were chosen, the MTBF for restored bearings would be 90 percent of the MTBF for brand new bearings, whereas the 10-percent life of restored bearings is 74 percent of the 10-percent life of new bearings.

If more realistic values of the parameters are used, say $\eta_t \leq L_{10}$ and $x > 1/2$, then the MTBF is within 3 percent of MTBF for the new bearings. Such a difference would be unnoticed in practice.

The problem of formulating a replacement policy, that is, a rule for making replacements in order to maximize reliability or minimize costs, can be based on the results and mathematical techniques presented herein. Actual policy must also be determined on the basis of economic conditions and logistical capabilities that have not been considered here. Therefore, it is beyond the scope of the present treatment to derive an optimum replacement policy. Nevertheless, the theory and results presented here have shown that the restoration process can return bearings to "almost new" life potential. The MTBF is not significantly affected and existing practices and replacement policies should not be adversely affected by the use of restored and refurbished bearings.

SUMMARY OF RESULTS

A life and reliability analysis for refurbished and restored bearings has been developed. In predicting the reliability of the restored bearing, it was assumed that the probability of survival of the bearing was equal to the product of the individual probabilities of survival for the newly stressed and previously stressed materials. Therefore, the stressed volume removed by grinding and the number of stress cycles accumulated before grinding are variables that affect the reliability of refurbished bearings. A sensitivity analysis using Lundberg-Palmgren theory and renewal theory was performed. The results of the analysis were as follows:

1. The mean time between failures (MTBF) is not significantly affected by the restoration parameters. The MTBF for refurbished and restored bearings ranges from 90 to 100 percent of MTBF for brand new bearings. Therefore, maintenance and replacement policies should not be adversely affected by the use of restored and refurbished bearings.
2. The 10-percent life of refurbished and restored bearings ranges from 74 to 100 percent of the life of brand new bearings. This life depends on the bearing's previous service time and the amount of material volume removed.
3. The failure distribution for restored and refurbished bearings is non-Weibull with a bias toward more early failures than that of a Weibull distribution.

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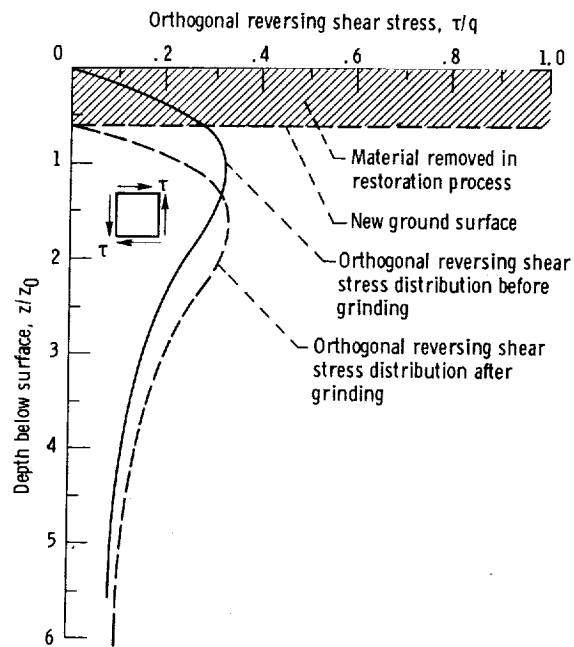


Figure 1. - Relative value of orthogonal reversing shear stress as function of depth below rolling-element surface. Figure shows effect of grinding in redistributing stress.

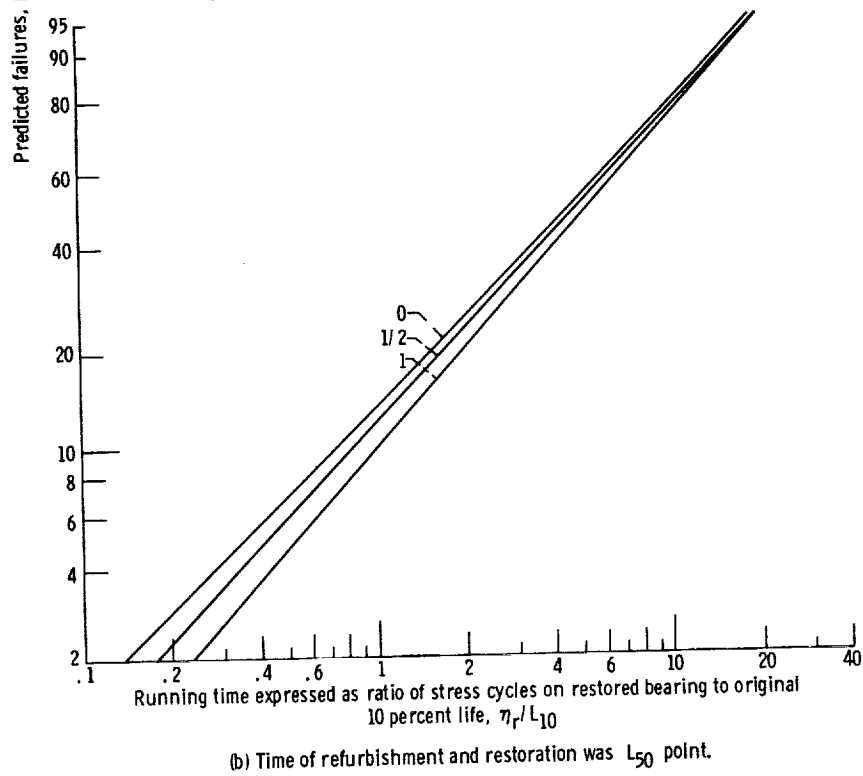
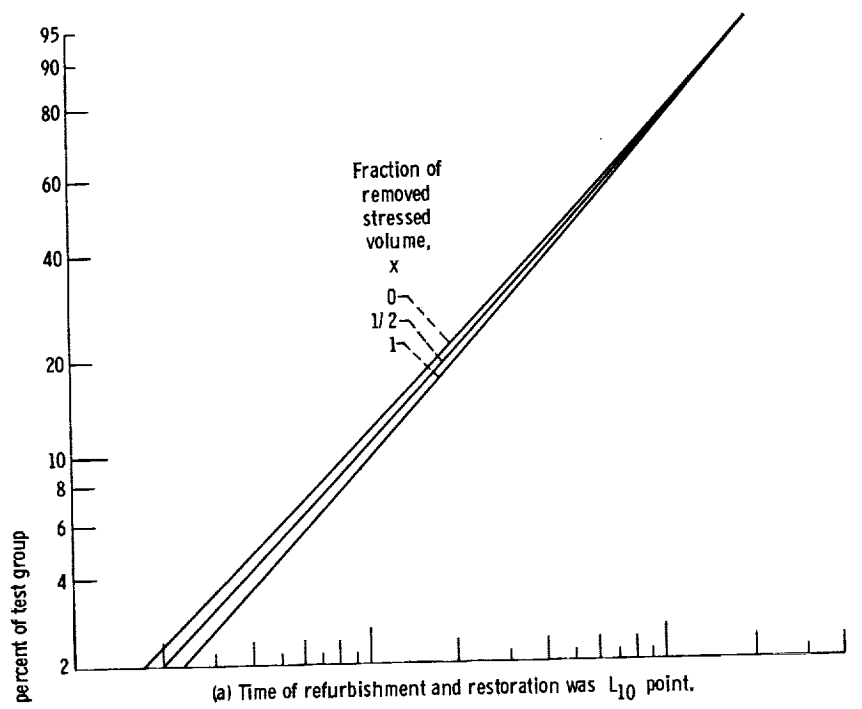


Figure 2. - Failure rate of refurbished and restored bearings for various fractions of stressed volume removed.

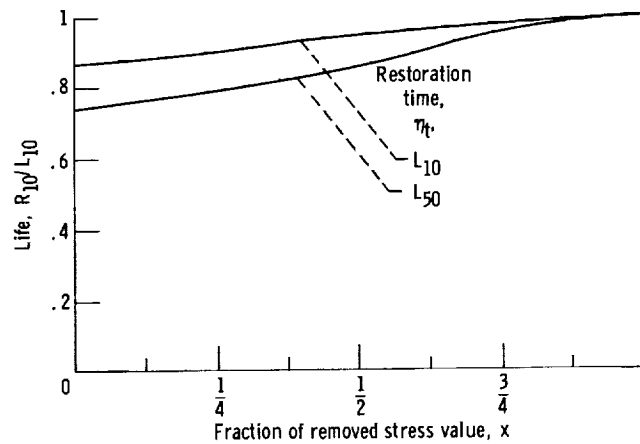


Figure 3. - Ratio of 10-percent life of restored bearing to 10-percent life of original bearings as a function of stressed volume removed in the refurbishing process. (Refurbishing at the 10-percent point gives greater restored life than using 50-percent point.)

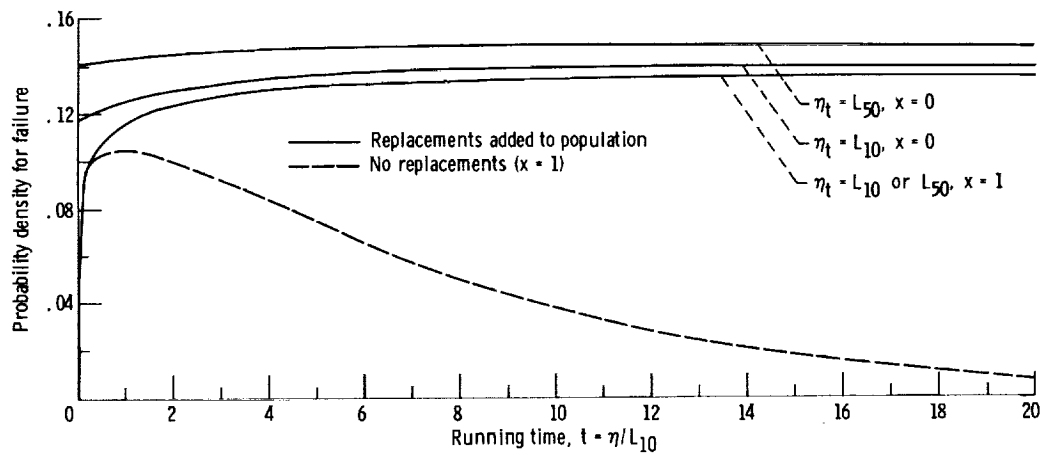


Figure 4. - Renewal density functions compared for refurbished, restored, and new bearings. Also shown is probability density for failure of new bearing assuming no replacements. Area under curves represents probability of failure.

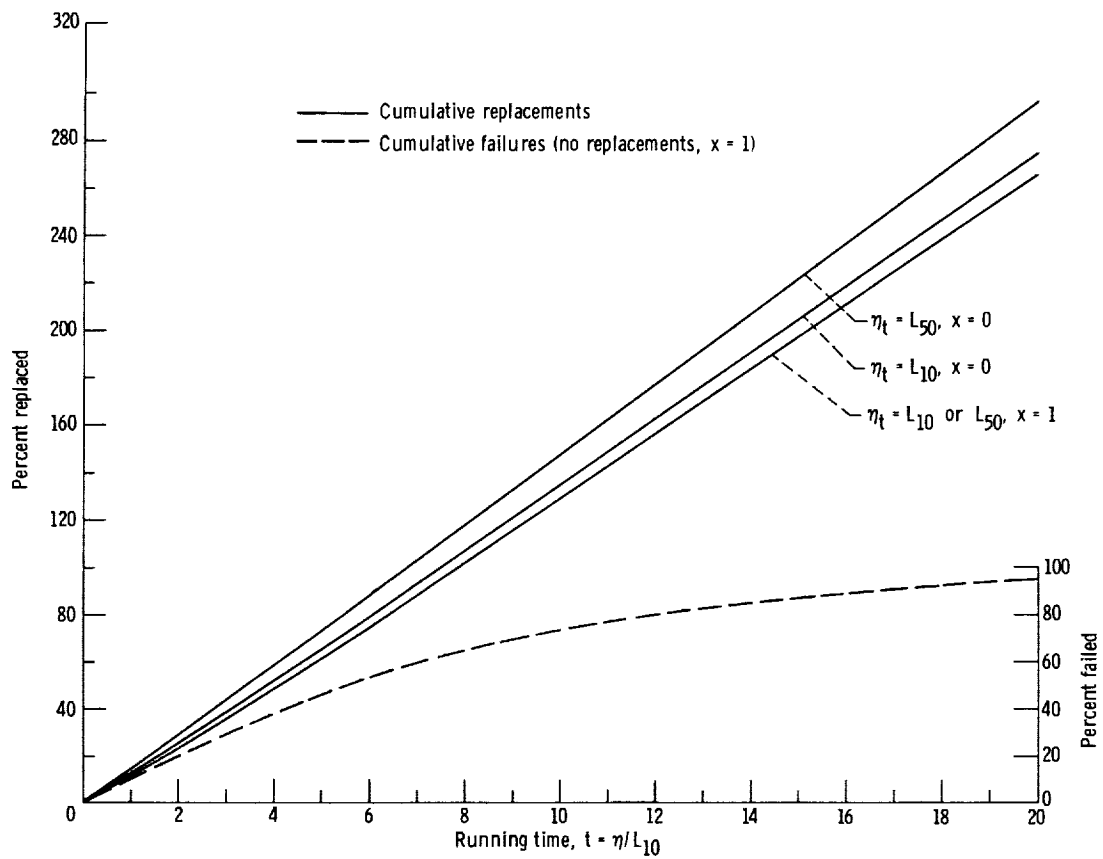


Figure 5. - Cumulative renewal and failure functions for refurbished, restored, and new bearings.

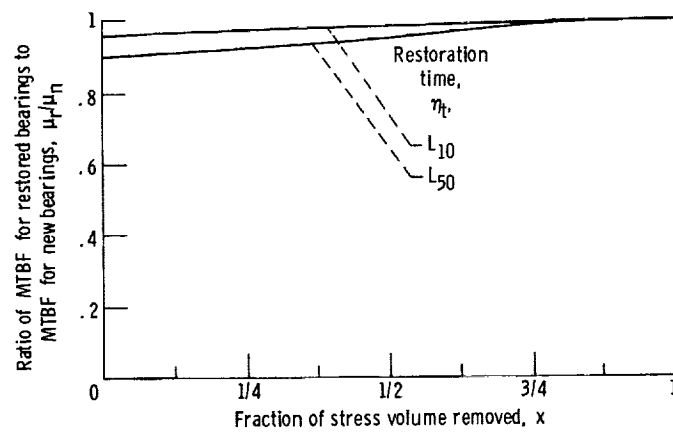


Figure 6. - Normalized mean time between failures (MTBF) as function of stressed volume removed in restoration process.